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ON CLIQUE-CLIQUE DOMINATION NUMBER AND INVERSE CLIQUE TRANSVERSAL NUMBER OF A GRAPH

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Abstract. Let $K(G)$ denote set of all cliques of a graph G. Two cliques $l_1, l_2 \in K(G)$ are said to *clique dominate* each other if there is a vertex common to both l_1 and l_2. A set $L \subseteq K(G)$ is said to be a *clique-clique dominating set* (CCD-set) if every clique in G is clique dominated by some clique in L. The clique-clique dominating set having minimum cardinality is called *clique-clique domination number* $\gamma_{cc}(G)$. In this paper, several bounds for the above parameter are obtained. Also bounds on number of cliques in a graph is given in terms of clique numbers.

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1. Introduction

All the graphs considered in this paper are finite, simple and undirected. For any undefined terminologies and notations refer [4, 15]. If a graph G is isomorphic to r copies of a graph H then we write it as $G \cong rH$. Two vertices are said to dominate each other if they are adjacent. A set $S \subseteq V$ is a dominating set if every vertex in $V - S$ is dominated by a vertex in S. The dominating set having minimum cardinality is called domination number $\gamma(G)$. These concepts of domination are well studied - see, for instance, [3, 6, 14]. Teffany et al. [13] have characterized the clique-dominating sets in the join, corona, composition, and cartesian product of graphs. They also determined the corresponding clique domination number of the resulting graph. Forbidden subgraph characterizations of graphs with a dominating clique or a connected dominating set of size three have been obtained by Margaret et al. [8]. The problem of dominating clique in interval graphs has been investigated by Sudhakaraiah et al. [11]. Interval graphs, due to their wide range of applications in the field of scheduling and genetics, have been taken into consideration for their investigation. As a result, certain classes of intervals have been considered, and their dominating cliques have been obtained. In their research, Mohanaselvi et al. [9] have successfully determined the exact values of the clique neighborhood domination numbers for various graph types, including the complete graph, complete bipartite

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graph, star graph, wheel graph, fan graph, banana tree, book graph, n-barbell graph, and friendship graph. For any $k, n \in \mathbb{Z}^+$ such that $n \leq 4$ and $1 \leq k \leq n$, Edward et al. [5] have shown that there exists a connected graph G, with $|V(G)| = n$ and clique secure domination number, $\gamma_{cls}(G) = k$. Additionally, they have established that for any $k, n, m \in \mathbb{Z}^+$ such that $1 \leq k \leq m$, there exists a connected graph G with $|V(G)| = n$, $\gamma_{cls}(G) = m$ and clique domination number, $\gamma_{cl}(G) = k$. Furthermore, they have presented the characterization of the clique-secure dominating set resulting from the join of two graphs.

The minimum number of vertices needed to cover all the edges of a graph is the vertex covering number $\alpha_0(G)$ while the independence number $\beta_0(G)$ is the maximum number of vertices in an independent set of G. These two numbers are related by classical Gallai's Theorem: $\alpha_0(G) + \beta_0(G) = p$. The upper vertex covering number $\epsilon(G)$ is the maximum order of a minimal covering of G. The independent domination number $i(G)$ is the minimum order of an independent dominating set of G. Naturally, we have an extension of Gallai's theorem to these numbers as: $\epsilon(G) + i(G) = p$.

A vertex $v \in V$ is a *cut-vertex* of a graph G, if $G - v$ is disconnected and such an edge is a *bridge* or a *cut-edge*. A graph G is *separable* if it has a cut-vertex otherwise it is *nonseparable*. A maximal nonseparable subgraph is a *block* of G. A maximal complete subgraph is a *clique*. The minimum number of cliques (not necessarily maximal) that cover all the vertices of a graph is well known in graph theory as partition number $\theta_0(G)$ introduced by Berge [1] and has been celebrated in Berge's conjecture on perfect graphs. Choudam et al. [2] studied its edge analogue edge clique covering number $\theta_1(G)$ defined as the minimum number of cliques that cover all the edges of a graph. The minimum number of colors needed to properly color the vertices of G is the chromatic number $\chi(G)$. Since independent sets and cliques exchange their properties on complementation $\theta_0(G) = \chi(\overline{G})$.

A vertex v is called *unicliqual* if it is incident on only one clique in G. If v is incident on more than one clique we call it a *polycliqual vertex*. We observe that $x = uv$ is contained in a clique l if both u and v are incident on l. If an edge is contained in a single clique, then the edge x is called *unicliqual*. If x is contained in more than one clique then x is called a *polycliqual edge*. In a cycle every vertex is bicliqual vertex and every edge is unicliqual. In corona $C_3 \circ C_3$ no edge is polycliqual edge. Every cutvertex is a polycliqual vertex. Every non cutvertex is unicliqual. In $K_4 - x$, the chord is a polycliqual edge.

Let $K(G)$ denote the set of all cliques of G. Let $P_C(G)$ denote the set of all polycliqual vertices of G. Let $|K(G)| = k$ and $|P_C(G)| = p_c$. Two cliques k_1 and k_2 are adjacent if there is a polycliqual vertex incident on k_1 and k_2. A *clique graph* $K_G(G)$ is a graph with vertex set $K(G)$ and any two vertices in $K_G(G)$ are adjacent if corresponding cliques in G have a vertex in common.

2. Bounds on number of cliques in a graph

The *minimum clique number* $\vartheta(G)$ is the order of a minimum clique of G while the *maximum clique number* $\omega(G)$ is the order of a maximum clique of G. It is immediate that $\vartheta(G) \leq \omega(G)$. Even though these two parameters are well studied in literature, the first parameter minimum clique number $\vartheta(G)$ received less attention and we are interested in it than the later. If G has an isolated vertex then $\vartheta(G) = 1$. If G is a triangle free graph without isolates, then $\vartheta(G) = 2$.

It is well known that an estimate for the number of edges in any graph is given by $\frac{p\delta}{2} \leq q \leq \frac{p\Delta}{2}$. In the following results we obtain an estimate for number of cliques in G. First consider the following results. A graph G is *clique regular* if every clique is of same order. Thus G is *r-clique regular graph* if $\omega(G) = \vartheta(G) = r$.

Proposition 2.1. *For any graph G with k cliques, edge clique covering number $\theta_1(G)$, and maximum clique number $\omega(G)$,*

$$\frac{2q}{\omega(\omega-1)} \leq \theta_1(G) \leq k. \tag{1}$$

Proof. Since a clique can cover at most $t = \frac{\omega(\omega-1)}{2}$ edges, we need at most $\frac{q}{t} = \frac{2q}{\omega(\omega-1)}$ cliques to cover all the edges of G and hence lower bound follows. Since the set of all cliques forms an edge clique covering of G, we have $\theta_1(G) \leq k$. □

Remark: For the Hajo's graph, $q = 9,\ \omega = 3,\ k = 4,\ \theta_1 = 3$.
We have $\frac{2q}{\omega(\omega-1)} = \dfrac{2 \times 9}{3 \times 2} = 3 = \theta_1(G) < 4 = k$.

For any Friendship graph F_3, $q = 9,\ \omega = 3,\ k = 3,\ \theta_1 = 3$.
Therefore, We have $\frac{2q}{\omega(\omega-1)} = \dfrac{2 \times 9}{3 \times 2} = 3 = \theta_1(G) = k$.

The Hajo's Graph and its complement is given below. In the Hajo's graph, the edges of the inner triangle are polycliqual edges while the remaining edges are unicliqual. In the compliment of Hajo's graph, all the edges are unicliqual edges.

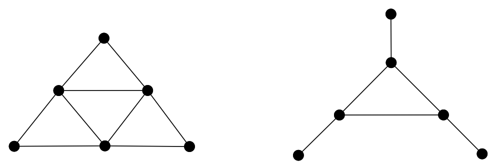


FIGURE 1. The Hajo's graph and its Complement

Corollary 2.2. *For any graph G with $\theta_1(G) = t$,*

$$\frac{t + \sqrt{t^2 + 8qt}}{2t} \leq \omega(G). \tag{2}$$

Proof. From Proposition 2.1, we have $\omega^2 t - \omega t - 2q \geq 0$. On solving this quadratic equation for ω we get the desired bound. □

For example, for the Hajo's graph, $\theta_1 = 3$, and $\frac{t+\sqrt{t^2+8qt}}{2t} = \frac{18}{6} = 3 = \omega(G)$.

On imposing certain conditions, an upper bound for number of cliques k in G is derived in the next result.

Proposition 2.3. *If every edge of G is unicliqual, then*

$$k \leq \frac{2q}{\vartheta(\vartheta - 1)}. \tag{3}$$

Proof. Since every edge of G is unicliqual, no two cliques share any edge of G. Further, any clique can contain at least $\binom{\vartheta}{2}$ edges. Hence $q \geq \binom{\vartheta}{2}k$. Then the result follows on simplification. The bound is sharp for any cycle C_n and any clique regular block graph. □

In the above proposition, condition that every edge is unicliqual is essential. For example, in Hajo's graph every edge is not unicliqual and one can check that the result (3) is not satisfied.

The above proposition suggests an upper bound for minimum clique number when the number of cliques k is known. Since the proof is similar to the proof of Corollary 2.2 we omit the proof of next corollary.

Corollary 2.4. *If every edge of G is unicliqual, then*

$$\vartheta \leq \frac{k + \sqrt{k^2 + 8qk}}{2k}. \tag{4}$$

The following characterization is straight forward.

Proposition 2.5. *Equality for k in Proposition 2.1 and 2.3 is attained if and only if G is a clique regular graph in which every edge of G is unicliqual.*

Corollary 2.6. *Equality for k in Proposition 2.1 and 2.3 is attained if G is a clique regular block graph.*

3. A note on minimum clique number

A graph G is cubic if every vertex is of degree 3. For a cubic graph $G \cong rK_4$, we have $\vartheta(G) = 4$. We prove that for any other cubic graph, $\vartheta(G)$ is always 2.

Proposition 3.1. *For any cubic graph $G \ncong rK_4$, $\vartheta(G) = 2$.*

Proof. Since $G \ncong rK_4$ and K_4 is the only cubic graph with $p < 6$, we assume that $p \geq 6$.
Case 1: Let G be a connected cubic graph with $p \geq 6$. Then $\vartheta(G) \leq 1 + \delta(G) = 1 + 3 = 4$. Also, Since G is connected, G has no isolates. Hence $\vartheta(G) \geq 2$.
Subcase 1.1: Suppose $\vartheta(G) = 4$. Then every clique is of order at least 4. Let $k_1 \in K(G)$ be any clique of order 4. Since G is connected, there exists at least one clique incident on some vertex say $u \in k_1$. But then $d(u) \geq 6$ - a contradiction.
Subcase 1.2: Suppose $\vartheta(G) = 3$. Then every clique is of order at least 3. If there exists a clique of order 4, we get a similar contradiction as in subcase 1.1. Therefore, we assume that every clique in G is of same order 3. Let $k_1 \in K(G)$ be any clique of order 3. Since G is connected, there exists at least one clique incident on some vertex say $u \in k_1$. Then $d(u) \geq 4$ - a contradiction. Hence we are forced to assume that there exists a $k_2 \in K(G)$ adjacent to k_1 such that k_1 and k_2 have a common edge uv. Let $\{w, u, v, x\}$ be the vertices of $k_1 \cup k_2$. Again, as G is connected and $p \geq 6$, there exists

a $k_3 \in K(G)$ adjacent to $k_1 \cup k_2$ such that $(k_1 \cup k_2) \cap k_3 = \{uv\}$ or $\{uw\}$ or $\{vw\}$ or $\{ux\}$ or $\{vx\}$. But then in any case we get $d(u) \geq 4$ or $d(v) \geq 4$ - a contradiction. Hence we have $\vartheta(G) \leq 2$. But we already have $\vartheta(G) \geq 2$. Therefore, we conclude that $\vartheta(G) = 2$.
Case 2: Let G be any cubic graph such that $G \ncong rK_4$. Then G has at least one component H such that H is a cubic graph with $p \geq 6$. Then by case 1, $\vartheta(G) = 2$. □

Since $\vartheta(G) = i(\overline{G})$, the following corollary is immediate.

Corollary 3.2. *For any cubic graph $G \ncong rK_4$, $i(\overline{G}) = 2$.*

As a more general result, we have the following characterization graphs for which $i(\overline{G}) = 2$.

Corollary 3.3. *For any graph G , $i(\overline{G}) = 2$ if and only if $\vartheta(G) = 2$.*

4. Clique-clique domination

Definition 4.1. Two cliques $k_1,\ k_2 \in K(G)$ are said to *clique dominate* each other if there is a polycliqual vertex incident with k_1 and k_2. A set $L \subseteq K(G)$ is said to be a *clique-clique dominating set (CCD-set)* if every clique in G is clique dominated by some clique in L. The cardinality of a minimum clique-clique dominating set is the *clique-clique domination number* $\gamma_{cc}(G)$.

It is immediate that $\gamma_{cc}(G) = \gamma(K_G(G))$.

Definition 4.2. A set $L \subseteq K(G)$is *cc-full* if every clique in L is adjacent to some clique in $K(G) - L$. The cardinality of a maximum cc-full set is the *cc-full number* $f_{cc}(G)$.

For the graph G with 15 cliques in Figure 2, $\gamma_{cc} = 4$ and $f_{cc} = 11$.

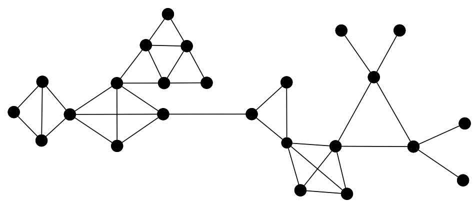


Figure 2. A graph G with 15 cliques

Proposition 4.3. *For any connected graph G with k cliques,*

$$\gamma_{cc}(G) \leq \left\lfloor \frac{k}{2} \right\rfloor. \tag{5}$$

Proof. Let L be a minimal CCD-set of G. Then $\gamma_{cc}(G) \leq L$. We also see that for any connected graph, if L is a minimal CCD-set of G, then $K - L$ is also a CCD-set of G. Hence $\gamma_{cc}(G) \leq |K(G) - L|$. Adding the above two equations we get,

$$\begin{aligned} 2\gamma_{cc}(G) &\leq |L| + |K(G) - L|, \\ &\leq |L| + |K(G)| - |L|, \\ &\leq |K(G)| = k. \end{aligned}$$

Then the result follows. □

Proposition 4.4. *For any connected graph G with k cliques,*

$$\gamma_{cc}(G) + f_{cc} = k. \tag{6}$$

Proof. Let L be a γ_{cc}-set of G. Then, $K(G) - L$ is a cc-full set of G. Hence $f_{cc} \geq |K(G) - L| = k - \gamma_{cc}$.
Therefore, $\gamma_{cc} + f_{cc} \geq k$.............(A)
On the other hand let D be a maximum cc-full set of G. Then $K(G) - D$ is CCD set of G.
Hence $\gamma_{cc} \leq |K(G) - D| = k - f_{cc}$.
Therefore, $\gamma_{cc} + f_{cc} \leq k$.............(B)
Then the result follows from (A) and (B). □

The *cc-degree*(clique-clique degree) of a clique l, $d_{cc}(l)$ is the number of cliques adjacent to l. Let $\Delta_{cc}(G)$ and $\delta_{cc}(G)$ denote the maximum and minimum cc-degrees of G respectively. In what follows, we use the following notations. For any clique $l \in K(G)$, *cc-neighbor* of l,

$$N_{cc}(l) = \{h \in K(G) | l \text{ and } h \text{ are adjacent}\}.$$

Proposition 4.5. *For any graph with maximum cc-degree Δ_{cc},*

$$\frac{k}{1 + \Delta_{cc}} \leq \gamma_{cc} \leq k - \Delta_{cc}.$$

Further, the bound is sharp.

Proof. Since a clique can cc-dominate at most Δ_{cc} cliques and itself, to cc-dominate k cliques we need at least $\dfrac{k}{1 + \Delta_{cc}}$ cliques. For the upper bound, let l be a clique of maximum cc-degree Δ_{cc}. Then $K(G) - N_{cc}(l)$ is a cc-dominating set of G. Hence $\gamma_{cc} \leq |K(G) - N_{cc}(l)| = k - \Delta_{cc}$. □

Any clique star and clique complete graph attain both upper and lower bounds.

5. Inverse Clique Transversal Number

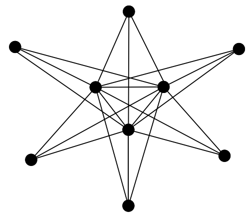
In 1990, the concept of vertex covering is extended as clique transversal number, defined and studied by Tuza [7], and later by Erdos et al. [10] in 1992.

5.1. Clique Transversal number.

A vertex $v \in V$ and a clique $h \in K(G)$ are said to cover each other if v is incident on the clique h. Minimum number of vertices that cover all the cliques of G is called *clique transversal number* $\tau_c(G)$. We immediately note that for any triangle free graph, $\alpha_0(G) = \tau_c(G)$.

A *windmill graph* $Wd(n, k)$ is a graph with k copies of complete graph K_n adjacent at a single vertex. In particular, $Wd(3, k) = F_k$ is called the *friendship graph*. Any windmill graph $Wd(n, k)$ is n-clique regular. A *generalized star* denoted $S(n, k)$ is a windmill graph in which each K_n has $n-1$ vertices in common. A generalized star is shown in Figure 3.

For any $v \in V$ the *open neighborhood* $N(v) = \{u \in V | u$ is adjacent to $v\}$ and the *closed neighborhood* $N[v] = N(v) \cup \{v\}$. Then degree $d(v) =$



Generalized Star S(4, 6)

FIGURE 3

$|N(v)|$. Let $\Delta(G)$ and $\delta(G)$ denote the maximum and minimum degree of G respectively. If $\overline{G}$ denote the complement of G then it is well known that

$$(*) \qquad \Delta(G) + \delta(\overline{G}) = \Delta(\overline{G}) + \delta(G) = p - 1.$$

The vc-degree is defined by Surekha Bhat et al. [12] The *vc-degree* (vertex clique degree) of a vertex u, $d_{vc}(u)$ is the number of cliques incident on u. Let $\Delta_{vc}(G)$ and $\delta_{vc}(G)$ denote the maximum and minimum vc-degrees of G respectively. We now obtain the following lower bound for clique transversal number.

Proposition 5.1. *Let G be any graph with k cliques and maximum vc-degree Δ_{vc}, then*

$$(7) \qquad \left\lceil \frac{k}{\Delta_{vc}} \right\rceil \leq \tau_c.$$

Further the bound is sharp.

Proof. Since any vertex can cover at most Δ_{vc} cliques, we need at least $\left\lceil \frac{k}{\Delta_{vc}} \right\rceil$ vertices to cover all the cliques of G. This yields the bound in (7).

Let G be any generalized star $S(n, k)$. Then $\tau_c(G) = 1 = \frac{k}{k} = \frac{k}{\Delta_{vc}}$. Also, if G is any clique cycle, then $\tau_c(G) = \left\lceil \frac{k}{2} \right\rceil = \left\lceil \frac{k}{\Delta_{vc}} \right\rceil$. Thus the bound is sharp. □

In the next proposition we obtain a condition under which a set D is an independent dominating set. For any vertices u and v, the subgraphs $\langle N[v] \rangle$ and $\langle N[u] \rangle$ are said to be *clique disjoint* if they have no clique in common.

Proposition 5.2. *For any graph G, a set $D = \{v_1, v_2, \ldots, v_k\}$ is an independent dominating set of G if and only if the following two conditions hold.*

(i) $\cup_{i=1}^{k} N[v_i] = V$.
(ii) $\langle N[v_i] \rangle \cap \langle N[v_j] \rangle$ *is clique disjoint for every* v_i, v_j, $1 \leq i \neq j \leq k$.

Proof. Suppose $D = \{v_1, v_2, \ldots, v_k\}$ is an independent dominating set of G. Then condition (i) holds as D is a dominating set of G. To show that condition (ii) holds. If possible assume that $\langle N[v_i] \rangle \cap \langle N[v_j] \rangle$ is not clique disjoint for every v_i, v_j, $1 \leq i \neq j \leq k$. Then there exists at least one clique $h \in \langle N[v_i]\rangle \cap \langle N[v_j]\rangle$. Hence $h \in \langle N[v_i]\rangle$ and $h \in \langle N[v_j]\rangle$ for some $v_i, v_j,\ 1 \leq i \neq j \leq k$. This implies that the clique h is incident on both v_i and v_j leading to the conclusion that v_i and v_j are adjacent - a contradiction.

Conversely, because of the condition (i), $D = \{v_1, v_2, \ldots, v_k\}$ is a dominating set of G. Suppose $\langle N[v_i]\rangle \cap \langle N[v_j]\rangle$ is clique disjoint for every $v_i, v_j,\ 1 \leq i \neq j \leq k$. Then the distance between any v_i and v_j is at least 2. Hence $D = \{v_1, v_2, \ldots, v_k\}$ is independent. Thus D is an independent dominating set of G.

□

We now define inverse clique transversal set as follows.

Definition 5.3. Let D be a minimum clique transversal set of an isolate free graph G. If $V - D$ also contains a clique transversal set D' of G, then D' is called an *inverse clique transversal set* with respect to D. The cardinality of a minimum inverse clique transversal set is the *inverse clique transversal number* $\tau_c'(G)$.

The necessary and sufficient condition for existence of at least one inverse clique transversal set of G is that G has no odd cycle C_{2n-1}.

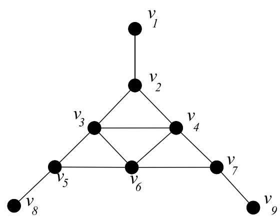


FIGURE 4. A graph G with $\tau_c' = 5$

Example 5.4. For the graph G of Figure 4, $\{v_2, v_5, v_7, v_4\}$ is a τ_c-set. Thus $\{v_1, v_8, v_9, v_3, v_6\}$ is a τ_c'-set. Hence $\tau_c = 4$ and $\tau_c' = 5$.

Proposition 5.5.

(1) If G is a clique cycle, then $\tau_c'(G) = \left\lceil \frac{k}{2} \right\rceil$.

(2) If G is a clique path, then

$$\tau_c'(G) = \begin{cases} \left\lceil \frac{k}{2} \right\rceil + 1, & k \equiv 0 (mod 3) \\ \left\lceil \frac{k}{2} \right\rceil, & otherwise \end{cases}$$

(3) For any generalized star, $\tau_c'(G) = 1$.

Proposition 5.6. *For any τ_c-invertible graph G with k cliques,*

$$\tau_c'(G) \leq k.$$

Proof. Let v_i be a unicliqual vertex in the clique $l_i,\ 1 \leq i \leq k$. Then $S = \{v_1, v_2, v_3, \ldots, v_k\}$ is an inverse clique transversal set of G. Therefore, $\tau_c'(G) \leq |S| = k$. □

Proposition 5.7. *Let T be a tree such that every non-end vertex is adjacent to at least one end vertex, then*

$$\tau_c(T) + \tau'_c(T) = p.$$

Proof. Let T be a tree. If every non-end vertex of T is adjacent to at least two end vertices, then the set of all non-end vertices is a minimum clique transversal set and the set of all end vertices is a minimum inverse clique transversal set. Suppose there are non-end vertices which are adjacent to exactly one end vertex. Let D and D' denote the minimum τ_c and τ'_c-set respectively. Let u be a non-end vertex adjacent to exactly one end vertex v. If $u \in D$, then $v \in D'$ and if $u \in D'$, then $v \in D$. In any case $|D| + |D'| = p$.

Thus $\tau_c(T) + \tau'_c(T) = p$. □

6. Conclusion

In this study, we delved into the well-established concepts of the minimum clique number and maximum clique number of a graph. Building upon this foundation, our work focused on deriving a series of bounds for these fundamental parameters. Additionally, we introduced a novel parameter called the "cc-domination number," presenting its formal definition and a comprehensive characterization of graphs that achieve this particular number. Throughout our investigation, we explored various properties of the "cc-domination number," shedding light on its intriguing characteristics.

This research contributes to a deeper understanding of graph properties and opens up new avenues for further exploration in the field of graph theory. By establishing connections between existing parameters and introducing a novel one, we hope to inspire future researchers to delve into the rich landscape of graph analysis and uncover even more intricate relationships within these mathematical structures. Ultimately, our findings add to the collective knowledge of graph theory and may find applications in diverse fields such as computer science, network analysis, and optimization.

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